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Investigating the Ability of Fractal Models to Estimate Retention Curve to Improve Water and Soil Resources Management

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Abstract

Soil water retention curve is among the most important properties needed for many soil and water management purposes. Due to high spatial and temporal variability in soils, its direct measurement is rather difficult, time consuming and expensive. Consequently, it would be more feasible to estimate it by using some indirect mathematical methods. The objectives of this investigation are to (1) determine the fractal dimension of the soil retention curve by fitting fractal models to the measurements and (2) investigate the relationship between the fractal dimension and other physical/textural/hydraulic parameters such assoil particle fractions of clay, silt, and sand in large scale. For this purpose, 190 soil samples with broad range of textures from four large agricultural areas were collected, and their particle size distribution, bulk density, organic carbon, salinity, pH, and retention curves were measured. To evaluate the performance of examined fractal models, three statistical parameters including RMSE, RMSD and R^2 were used. Results indicated that the fractal dimension has an inverse relationship with soil texture; the finer the soil texture, the greater the fractal dimension. The lowest and greatest fractal dimensions of the Tyler-Wheatcraft model in loamy sand and clay textures were obtained to be 2.38 and 2.74, respectively. These were significant at 1% level based on the Duncan’s multiple range tests. Results further showed that the most accuracy of estimating retention curve in different soil textures by using van Genuchten, Brooks-Corey, and Tyler-Wheatcraft with normalized errors average obtained were 0.06, 1.09, and 3.27, respectively. Furthermore, the obtained R^2 values were ranged from 0.88 to 0.99 for Tyler-Wheatcraft and van Genuchten models, respectively. Compares to Brooks-Corey model, the van Genuchten retention model provided better accuracy in estimating retention curve for different soil textures.

Keywords: Fractal Geometry; Retention Curve; Soil Hydraulic Functions.

INTRODUCTION

Soil is a complex system which plays a very significant role in environment and agriculture and is one of the basic parts of natural ecosystem (Veltri et al., 2013). Correct estimation of soil hydraulic parameters is the most important factor in modeling water movement and solute transport in soil. Soil water retention curve and unsaturated hydraulic conductivity are fundamental information for investigating the movement of water, solutes, and pollutants in root zone that their quantitative expressions are vital for implementation of best management practices. These two functions are very important and provide useful information on soil physical properties and water and soil management at various scales.

Both soil texture and soil structure influence the retention curve (Hillel, 1998). Direct measurement of soil hydraulic function is expensive, time consuming and tedious. Consequently, there has been a tendency to make serious efforts to estimate them by some indirect methods. Several empirical models were then introduced in the literature to quantitatively express soil retention (e.g. Burdine (1953), Brooks and Corey (1964), Brooks and

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Corey (1966), Campbell (1974), van Genuchten (1980)). On the other hand, all hydraulic characteristics of soil and water properties, hydraulic conductivity, and infiltration have a close relationship with the geometry of porous materials (Ghorbani Dashtaki et al. 2010, Khodaverdilooet al. 2011). Porous environments including the soils are non-isotropic systems with several different components which make the simulations of their hydraulic properties very complex (van Damme, 1958, Khodaverdiloo et al. 2011). Thus, soil hydraulic characteristics can be estimated based on soil particle size (Millan et al. 2003, Babaeian et al. 2016). In the last two decades, fractal method has been applied as a suitable tool for modeling complex phenomena. Fractal geometry has provided new ideas for quantitative description of non-isotropic porous environments such as soils. Fractal geometry is a complex approach which has been used repeatedly in wide areas all over the world at different scales. Consequently, due to spatial variability of soils it has been suggested to use the fractal geometry for soils that demonstrate different hydraulic behavior at different scales (Dirksen 1991, Eghbal et al. 1993, Seuront et al. 1999, Lee, 2002, Lin, 2008). Fractal dimension is a potential parameter for scaling that has the capability of measuring the environmental chaos degree (Turcotte 1986, Wu et al. 1993, Wang et al. 2018).

The application of fractal methods has been led to presenting some quantitative functions to predict soil water retention curve, unsaturated and saturated hydraulic conductivity, as well as pore size distribution. It has been reported that fractal theory can model soil structure and is able to link non-uniform soil structure to specific soil behavior (Young et al. 2001). Some investigators have used fractals for quantitative expression of the relationship between soil and water (Perrier et al. 1996, Giménez 1997), estimating retention curve, infiltration (Perfect and Kay 1991, Perfect et al. 1992), and water movement in soil (Crawford 1994, Kosugiand Hopmans 1998, Tylerand Wheatcraft 1990, 1992). The so-called scaling is a method in which by using a scaling factor the connection among the characteristics of different soils are studied and connected to each other. The scaling methods have originally developed based on similar media theory. In these methods, hydraulic functions of different soils have the capability to transpose with certain quantities of physical length (scale factor) on each other and be shown by a reference curve (Warrick 1977).

Some researchers have reported that the retention curve follows almost the same shape as aggregate size distribution curve. Consequently, the aggregate size distribution models are suitable estimators of soil retention curve (Lee and Ro 2014). So aggregate size distribution models are divided into two general groups of fractal and non-fractal models. Tyler-Wheatcraft (1990) and van Genuchten (1980) can be mentioned as fractal and non-fractal models, respectively.

Assouline et al., (1997) found that soil pH, organic matter, and cation exchangeable capacity have an effect on soil compaction, and therefore soil retention curve will also change. Therefore, retention curve must be determined again after compaction which is time consuming and costly. Zhang et al., (2006) investigated the effect of three different levels of compaction on hydraulic properties in two samples of silty loam soils and found that soil compaction affects its soil hydraulic properties greatly. Tuli et al. (2001) used joint scaling factors by combining methods presented by Kosugi and Hopmans (1998) based on Miller and Miller (1956) theory for scaling retention curve and soil hydraulic conductivity; however, this method can only be applied for environments of similar geometry. By studying fractal essence of different soil textures, Millan et al., (2003) expressed that fractal dimension of particle size distribution has a significant linear correlation with clay content in a way that by increasing clay percentage, the fractal dimension increases, whiledecreases by increasing sand percentage.

Optimum retention curve has a physical concept but its direct measurement is difficult and time consuming both in field and laboratory. One of the useful methods for estimation of suitable retention curve (fractal dimension) is using porosity size distribution with the help of pictures analyses (Bartoli et al. 2005, Rasiah et al. 1992, 1993).

Veltri et al., (2013) analyzed retention curve by using fractal geometry. In their study, the relationship between retention curve and fractal dimension has been investigated and it has been found that fractal scaling of retention curve makes the measurement of soil water under unsaturated conditions possible. Souto et al., (2015) evaluated fractal models for estimating hydraulic functions in unsaturated soils, which indicated that the new equation is able to estimate soil moisture data in four soil series with different textures.

In last two decades, fractal methods were used as a new tool for modeling soil and water-related issues and some functions were suggested to account for soil retention curve, unsaturated hydraulic conductivity, and pore size distribution. In this research we attempt to estimate soil hydraulic parameters and present a practical relationship to express soil retention curve by a fractal model. Consequently, the objective of this study was to obtain the fractal dimensions of soil retention curves of widely different soil textures by using fractal models. Furthermore, it was aimed to come up with a simple relationship to relate the fractal form of van Genuchten retention model with soil particle size distribution.

MATERIALS AND METHODS

Soil Sampling

A number of 190 soil samples with widely different textures were collected from top soils of widely different areas and delivered to laboratory to accomplish the designated soil analyses. The designated soil physical and chemical properties were measured, using standard methods. The soil texture, organic matter content, electrical conductivity, and soil pH were measured by using hydrometer method, Walkley and Black method, EC meter, and pH meter, respectively (Arabi et al. 2017). The soil water retention curves of all soil samples were obtained by using a pressure plates apparatus. The van Genuchten parameters of measured retention curves were obtained by using the RETC program (van Genuchten et al. 1991). The mean values of some physical characteristics of studied soils are presented in Table 1. Data presented in this table shows that the studied soil samples are classified into 11 classes among 12 soil texture classes. Such a wide soil texture sampling was performed to assess the capability of fractal models in a very wide range of soil particle size distributions. According to Table 1, the studied soil samples have the porosity of 0.35 to 0.57. The minimum and maximum bulk density ranged from 1.12 to 1.70 g/cm^3 in the studied soil samples. The distribution of all studied soil textures is presented in Fig 1. This figure shows that the experimental soil textures cover most texture classes ranging from sandy to clay soil textures and dissimilar to most previously conducted studies do not cover only one or two textural classes.

A fractal model of Tyler and Wheatcraft (1990) was used to obtain the fractal dimensions of the retention curves. To derive a pore size-based model, Tyler and Wheatcraft (1991) have used the Sierpinski carpet pattern introducing a power form model to account for the retention models of Brooks and Corey (1964) and Campbell (1974). Their proposed model can be written as:

Table 1. The mean values of some physical characteristics of studied soils

Soil Texture	Number of soil samples	Average ± Std.			Min-Max			Average ± Std.		Min-Max	
		Sand	Silt	Clay	Sand	Silt	Clay	Porosity	Bulk density	Porosity	Bulk density
Sandy	7	94±2.3	3±1.3	3±2.1	90-96	1-5	1-7	0.40±0.04	1.58±0.10	0.36-0.46	1.40-1.72
Loamy Sand	9	83±4.9	10±7.3	8±3.3	73-88	1-20	2-12	0.42±0.04	1.53±0.11	0.38-0.48	1.35-1.62
Sandy Loam	37	68±6.2	21±8.7	11±5.0	57-84	2-36	1-19	0.44±0.06	1.51±0.15	0.30-0.55	1.15-1.85
Sandy Clay Loam	21	59±7.3	17±7.0	24±4.1	46-71	5-27	20-33	0.43±0.06	1.51±0.18	0.33-0.54	1.10-1.77
Loam	13	39±6.0	41±5.8	20±4.8	32-49	28-48	12-26	0.64±0.17	1.10±0.43	0.40-0.92	0.46-1.80
Silt	11	8±4.5	87±4.3	5±2.9	1-15	80-95	1-11	0.47±0.05	1.39±0.14	0.39-0.56	1.13-1.69
Silty Loam	21	19±9.8	66±9.1	15±7.7	1-34	51-81	2-26	0.46±0.07	1.42±0.18	0.35-0.65	0.91-1.66
Clay Loam	14	34±6.6	33±6.2	33±5.7	23-44	23-45	19-39	0.50±0.05	1.33±0.10	0.45-0.63	1.07-1.43
Silty Clay Loam	16	11±6.1	56±7.5	34±4.9	1-20	41-70	27-43	0.49±0.04	1.34±0.13	0.40-0.55	1.14-1.66
Sandy Clay	8	55±3.9	6±3.8	39±3.0	50-60	1-12	36-46	0.40±0.05	1.59±0.14	0.36-0.46	1.40-1.72
Silty Clay	7	12±6.8	45±4.4	43±3.8	1-24	36-48	40-51	0.48±0.06	1.39±0.18	0.39-0.57	1.16-1.64
Clay	26	20±12.3	20±12.1	60±11.9	2-44	1-38	43-85	0.51±0.12	1.28±0.29	0.37-0.79	0.54-1.67
Total	190	41±26.8	33±24.0	25±18.6	1-96	1-95	1-85	0.47±0.10	1.41±0.24	0.30-0.92	0.46-1.85

$$\theta = \theta_s \left(\frac{h}{h_0}\right)^{D_m - 3} \quad (2)$$

in which n and m are shape parameters of retention curve, α (L^{-1}) approximately equals to inverse of bubbling pressure, h (L) is soil water pressure head, θ_r ($L^3.L^{-3}$) and θ_s ($L^3.L^{-3}$) are the residual and saturated soil water contents ($L^3.L^{-3}$), respectively. where h_0 is the bubbling pressure, θ_s is the saturated water content, and D_m is the fractal dimension. The RETC program was used to obtain the parameters of retention curve.

Model performance

In order to compare the results of fractal models with the measured data, three statistics including normalized Root Mean Square Error (nRMSE), Root Mean Square Deviation (RMSD) and determination of coefficient (R^2) were used. The relationships can be expressed as (Zarei et al. 2010, Alfaro Soto et al. 2017):

$$nRMSE = \frac{\sqrt{\sum(\theta_p - \theta_m)^2 / n}}{\bar{\theta}_m} \quad (4)$$

$$RMSD = \sqrt{[\log(\theta_m) - \log(\theta_p)]^2 / (n-1)} \quad (5)$$

$$R^2 = \frac{\left(\sum(\theta_p - \bar{\theta}_p)(\theta_m - \bar{\theta}_m)\right)^2}{\left(\sum(\theta_p - \bar{\theta}_p)^2 (\theta_m - \bar{\theta}_m)^2\right)} \quad (6)$$

in which θ_m is the measured soil water content, $\bar{\theta}_m$ is the mean of measured soil water content, θ_p is the estimated water content, and n is the number of studied soil samples.

In addition, the one-way ANOVA analyses at 1% and 5% of significance levels were used. All statistical analyses were performed by SAS software.

RESULTS AND DISCUSSIONS

The obtained fractal dimensions of Tyler and Wheatcraft (1990) model are presented in Table 2 for all examined soil textures. Data presented in this table show that the fractal dimensions have an inverse relationship with soil texture. As such, the finer the soil texture, the larger fractal dimension value. The lowest and largest values of fractal dimensions of Tyler-Wheatcraft model in loamy sand and clay textures were obtained to be 2.38 and 2.74, respectively. Results given in Table 2 further indicate that the fractal dimension of Tyler-Wheatcraft model demonstrates higher variability than classic model; as such, the minimum and maximum values of this model were varied between 2.89 to 2.92.

The correlation between the soil particle size distribution and the obtained fractal dimensions is presented in Table 3. As can be followed from this table, the sand percentage with Tyler-Wheatcraft and the clay content with classic models correlate well at 1% significant level. Results presented in Table 3 further indicate that based on Duncan's multiple range tests; there is a significant correlation between the particle size distributions with Tyler-Wheatcraft fractal dimensions at 1% level. Table 4 represents the calculated statistics for comparison of measured and estimated retention curves obtained with different models. The presented data in Table 4 further indicate that maximum and minimum precision of estimated retention curve of the examined soil samples were obtained, respectively, by van Genuchten and Tyler-Wheatcraft models for loam texture. Furthermore, the highest accuracy was obtained by van Genuchten, Brooks-Corey, and Tyler-Wheatcraft models with average normalized errors of 0.06, 1.09, and 3.27, respectively. The calculated values of coefficients of determination were, respectively, ranged from 0.88 to 0.99 in Tyler-Wheatcraft and van Geuchten models. Considering the obtained results, van Genuchten model has the highest accuracy for estimating retention curve of different soil textures. Parameters of van Genuchten model for different soil textures are presented in Table 5. The data presented in this table show that the smaller the size of soil particles, the lower m parameter value. So that, the minimum value of this parameter obtained for loam texture is 40. Figure 2 shows the retention curve of soil samples with different textures drawn by Brooks-Corey (BC), van Genuchten (VG), and Tyler-Wheatcraft (TW) models. Considering the large number of obtained figures for all soil samples, only one sample for each soil texture is presented. Figure 2 further shows that van Genuchten model has the highest accuracy in estimating retention curve of different soil textures. Furthermore, this figure demonstrates that Brooks-Corey and Tyler-Wheatcraft fractal models have no significant difference in estimating retention curve. Figure 3 depicts the measured against estimated soil water content by using van Genuchten, Brooks-Corey, and Tyler-Wheatcraft models. It can be followed from this figure that van Genuchten model provides the most accurate estimation of water content among other studied models.

Table 2. The obtained fractal dimensions of Tyler-Wheatcraft model

Soil Texture	Number of soil samples	Tyler-Wheatcraft Model		
		Min	Average ± Std.	Max
Sandy	7	2.41	2.42±0.01	2.43
Loamy Sand	9	2.42	2.44±0.02	2.47
Sandy Loam	37	2.44	2.54±0.03	2.50
Sandy Clay Loam	21	2.47	2.50±0.02	2.53
Loam	13	2.52	2.54±0.01	2.86
Silt	11	2.60	2.61±0.01	2.63
Silty Loam	21	2.55	2.59±0.02	2.63
Clay Loam	14	2.52	2.55±0.02	2.58
Silty Clay Loam	16	2.53	2.60±0.03	2.63
Sandy Clay	8	2.50	2.51±0.01	2.52
Silty Clay	7	2.58	2.61±0.02	2.63
Clay	26	2.53	2.59±0.03	2.65
Total	190	2.41	2.54±0.06	2.65

Table 3. Correlation between soil particle fractionsand fractal dimensions

Fractal Model	Sand (%)	Silt (%)	Clay (%)
fractal dimension of Tyler-Wheatcraft (D_m)	0.961**	0.500	0.250

** significant at 1% levelbased on Duncan's multiple range test.

Table 4. The obtained statisticsto assess the performance of examined models

Classification of Soil Texture	van Genuchten			Brooks-Corey			Tyler-Wheatcraft		
	R^2	RMSD	NRMSE	R^2	RMSD	NRMSE	R^2	RMSD	NRMSE
Sandy	0.99	0.10	0.11	0.99	0.13	0.27	0.99	0.13	0.54
Loamy Sand	0.99	0.04	0.04	0.99	0.06	0.07	0.99	0.07	0.16
Sandy Loam	0.99	0.03	0.04	0.99	0.06	0.27	0.99	0.05	0.49
Sandy Clay Loam	0.99	0.03	0.09	0.98	0.08	1.42	0.98	0.08	1.64
Loam	0.99	0.00	0.00	0.93	0.13	4.84	0.88	0.14	11.82
Silt	0.98	0.02	0.10	0.98	0.06	1.32	0.97	0.06	1.85
Silty Loam	0.98	0.02	0.10	0.98	0.06	1.32	0.97	0.06	1.85
Clay Loam	0.98	0.02	0.10	0.95	0.11	3.61	0.92	0.12	7.54
Silty Clay Loam	0.98	0.02	0.11	0.97	0.09	1.95	0.97	0.09	2.06
Sandy Clay	0.99	0.01	0.01	0.96	0.08	1.99	0.96	0.08	2.04
Silty Clay	0.98	0.02	0.07	0.95	0.08	2.74	0.94	0.08	3.69
Clay	0.99	0.01	0.04	0.95	0.07	2.46	0.94	0.08	4.16
Total	0.99	0.03	0.06	0.97	0.09	1.90	0.96	0.09	3.27

Table 5. Mean of van Genuchten model parameters for different soil textures

Soil Texture	Number of Samples	α [1/cm]	n	m	θ_r [cm^3/cm^3]	θ_s[cm^3/cm^3]
Sandy	7	0.01	2.23	0.21	0.001	0.39
Loamy Sand	9	0.03	1.66	0.35	0.000	0.40
Sandy Loam	37	0.02	1.79	0.36	0.002	0.42
Sandy Clay Loam	21	0.01	4.27	1.43	0.018	0.47
Loam	13	0.00	1.00	1.27	0.051	0.47
Silt	11	0.01	1.18	0.40	0.000	0.50
Silty Loam	21	0.01	1.18	0.40	0.000	0.50
Clay Loam	14	0.00	0.83	14.00	0.062	0.50
Silty Clay Loam	16	0.00	0.97	0.81	0.021	0.49
Sandy Clay	8	0.00	0.88	1.17	0.069	0.51
Silty Clay	7	0.00	1.41	0.28	0.000	0.49
Clay	26	0.00	0.91	6.49	0.049	0.56

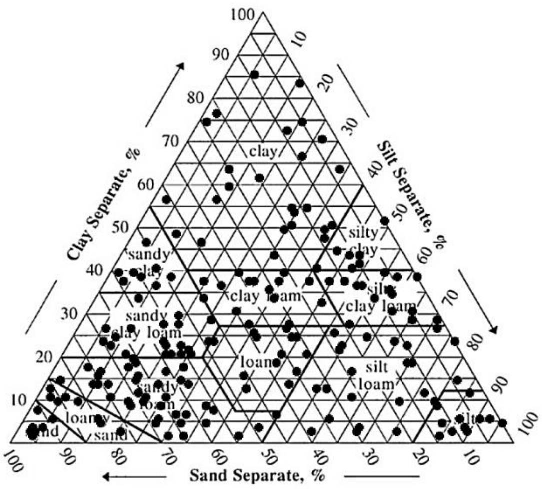
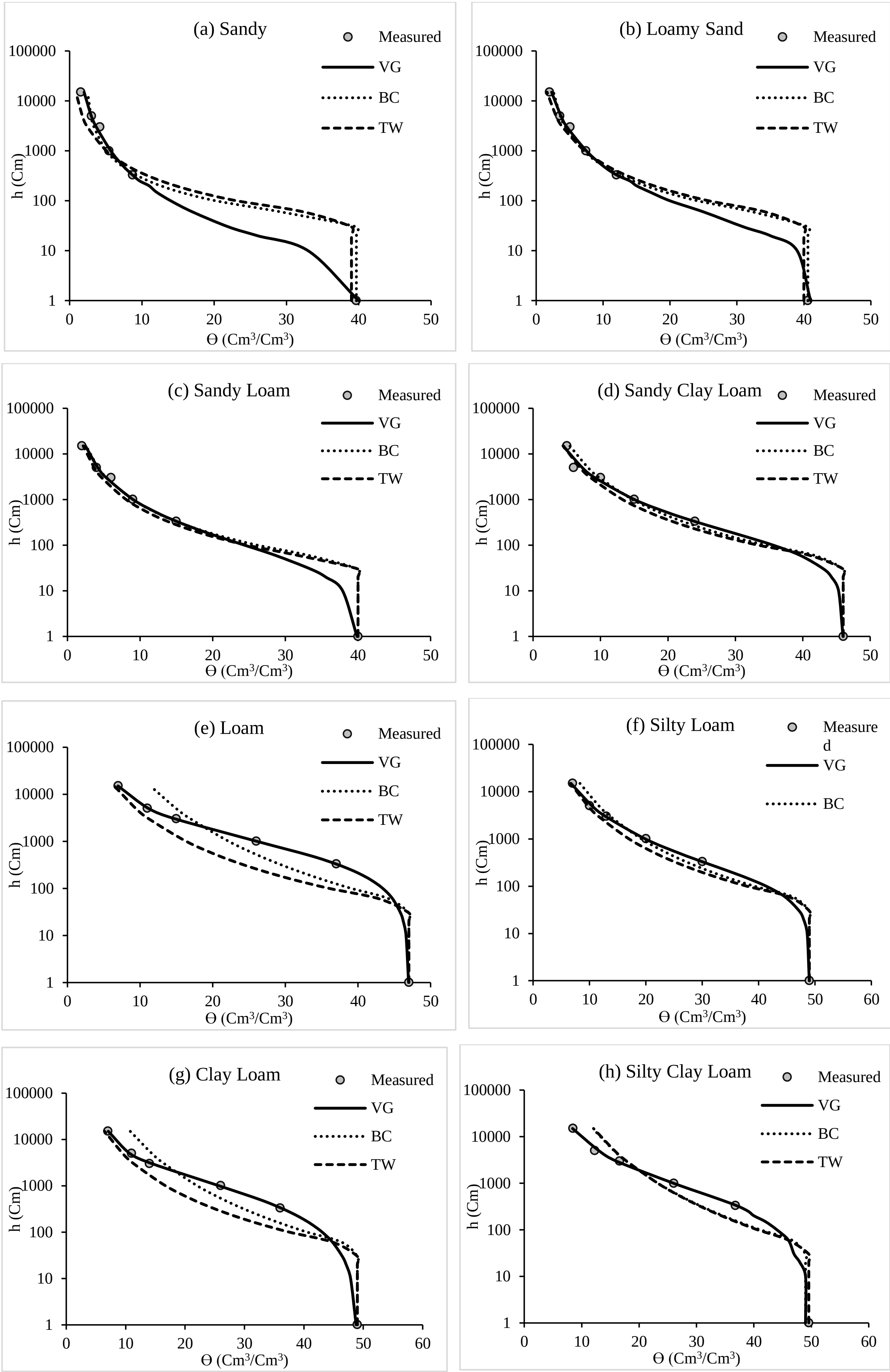


Fig. 1. The distribution of studied soil textures.


(a) Sandy
(b) Loamy Sand
(c) Sandy Loam
(d) Sandy Clay Loam
(e) Loam
(f) Silty Loam
(g) Clay Loam
(h) Silty Clay Loam
Measured
VG
BC
TW
h (Cm)
Θ (Cm³/Cm³)

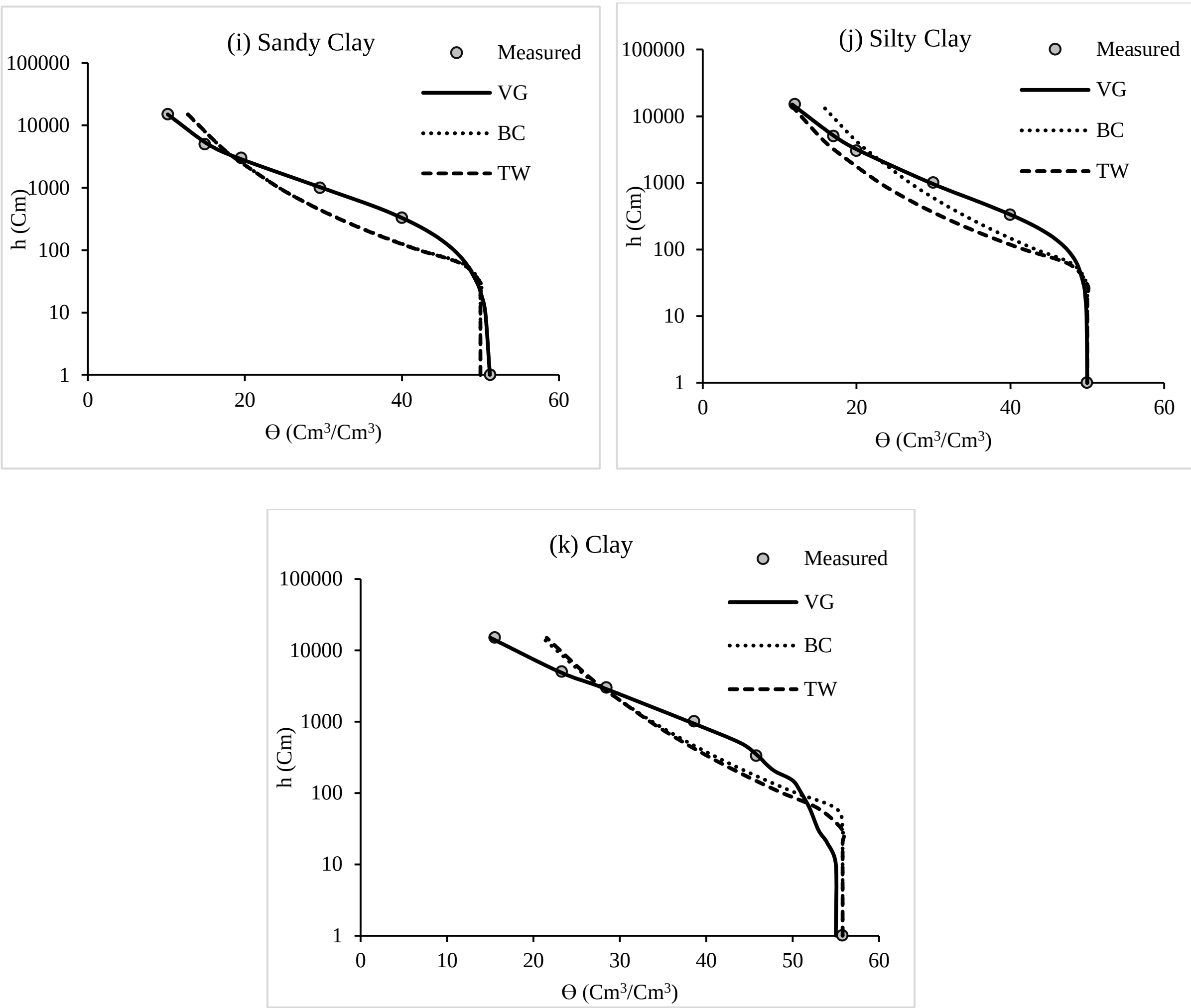
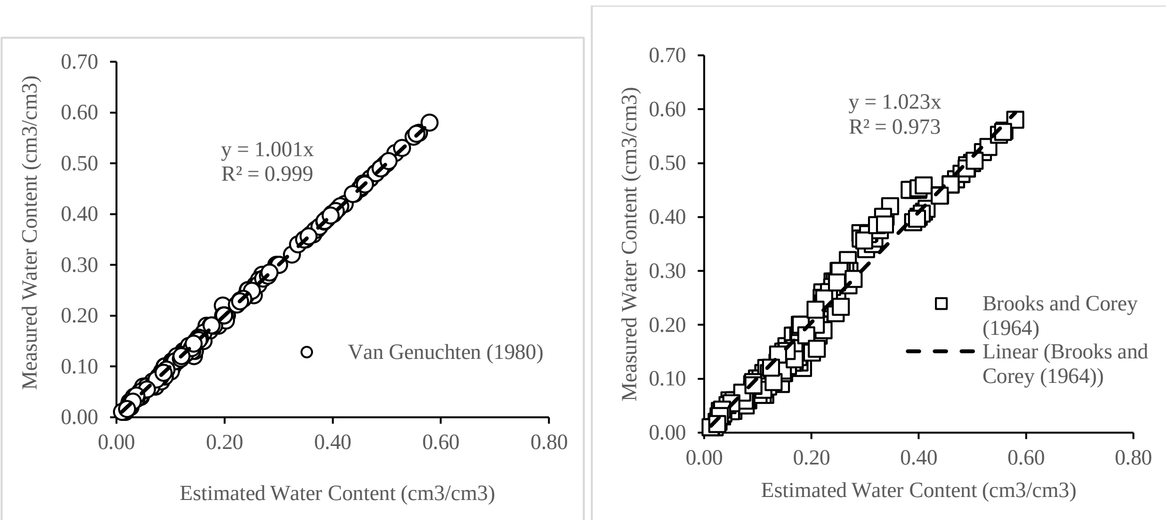


Fig. 2. The measured and predicted retention curves by different models for different soil textures.



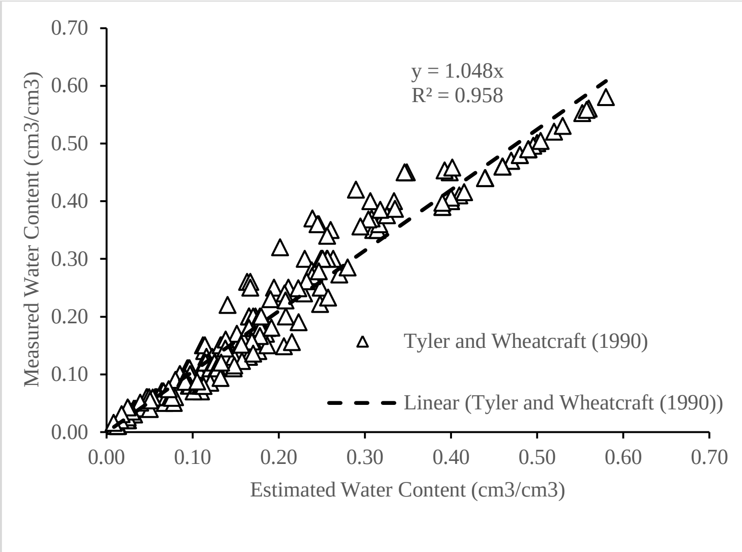


Fig. 3. Measured against thefittedsoil water contentwithfractal models.

CONCLUSIONS

Our results indicate that the magnitude of fractal dimensions inversely reflects the soil particle size distribution. As such, the smaller the soil sizes, the larger the fractal dimensions. The lowest (2.38) and largest (2.74) dimension values of Tyler-Wheatcraft fractal model obtained for loamy sand and clay textures, respectively. Furthermore, the most accurate point estimations of retention curve for different textures were obtained by using van Genuchten, Brooks-Corey, and Tyler-Wheatcraft models with average normalized errors of 0.06, 1.09, and 3.27, respectively. The obtained coefficients of determination vary from 0.99 to 0.88 for van Genuchten and Tyler-Wheatcraft methods, respectively. Among the studied retention models, the van Genuchten model provides the highest accuracy in estimating the retention curves of widely different soil textures.

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